A Quadrature-Hybrid Design Using a Four-Port Elliptic Patch

Kin-Lung Chan, Fayez A. Alhargan, and Sunil R. Judah

***Abstract*— Available designs of quadrature hybrids have used transmission lines, and recently, circular-disk patches. This paper introduces a four-port quadrature hybrid using a microstrip elliptic patch. It is shown that this configuration has good performance as a quadrature hybrid over a fairly large bandwidth. The performance is improved by the introduction of matching networks. The analysis is carried out using the cavity model Green's function with appropriate corrections for the fringing fields. A comparison of experimental and theoretical results show good agreement.**

***Index Terms*—Elliptic microstrip path, quadrature hybrid.**

I. Introduction

The quadrature hybrid, or branched-line coupler, has been one of the mainstays in microwave systems having uses as power splitters and mixers. The conventional $|S_{11}| < -20$-dB bandwidth is generally less than 5%. To increase the usable bandwidth of this junction, [1] introduced the equivalent admittance technique using stubs or quarter-wave transformers. Despite these additions the conventional branched-arm hybrid suffers from limited bandwidth and, in addition, the length of the quarter-wave sections become difficult to control accurately at the higher frequencies. To overcome this problem, researchers have turned to planar circuits [2], [3]. In [3], multimode operation and matching networks were used to improve the performance of the four-port circular-disk quadrature hybrid. Unfortunately this device has equal power split between the coupled waves only at a particular frequency and becomes rapidly unequal as one moves away from the design frequency, resulting in a limited bandwidth. Recently, [4], [5], and [6] introduced open-circuit stubs as dummy ports at the periphery of the disk in addition to the matching networks at the physical ports. This approach allows one to change the overall impedance matrix by suitable choice of the stub parameters which results in a quadrature hybrid with flat-coupling response and operational bandwidth of over 20%. At higher frequencies though, the length of the open circuit stubs become so short that it will be submerged by the fringing fields of the disk itself. In this paper it is shown that one can obtain good quadrature-hybrid performance without any dummy ports at the periphery by using an elliptic patch and impedance transformers.

The elliptic-patch antenna has been analyzed using numerical techniques, such as the moment method [7], the contour integral method [5], and spectral domain techniques [8]. Others have used an approximation on the basis of single mode excitation [9]. A closed-form expression was developed in [10], based on the cavity-model Green's function, for the impedance matrix of the elliptic patch. This expression is very computationally efficient and thus makes interactive design feasible. The quadrature hybrid is designed using the elliptic disk at the basic junction as shown in Fig. 1. By suitable choice of the eccentricity of the elliptic patch, a flat-coupling response is obtained for the coupled ports. Overall match over a bandwidth is then taken care of by stepped-impedance sections connected to the physical ports.

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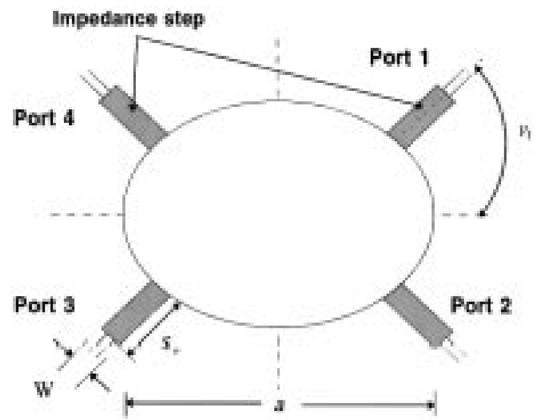

Fig. 1. Configuration of the elliptic-patch quadrature hybrid with impedance step-matching network.

II. Impedance Matrix for the Elliptic Patch

The Green's function of an elliptic patch consists of even and odd parts. So from [10], the even and odd elements of the impedance matrix Z for an elliptic patch with ports at the periphery (Fig. 1) are given as

$$Z_{ij}^{e} = \frac{j\omega\mu\, dl^2}{W_i W_j} \sum_{n=0}^{\infty} \cdot \frac{Je_n(h, \cosh u_1) ISe_n(h, v_i, \Delta_i, u_1) ISe_n(h, v_j, \Delta_j, u_1)}{Me_n(h) Je_n'(h, \cosh u_1)} \tag{1}$$

$$Z_{ij}^{o} = \frac{j\omega\mu\, dl^2}{W_i W_j} \sum_{n=1}^{\infty} \cdot \frac{Jo_n(h, \cosh u_1) ISo_n(h, v_i, \Delta_i, u_1) ISo_n(h, v_j, \Delta_j, u_1)}{Mo_n(h) Jo_n'(h, \cosh u_1)} \tag{2}$$

where $Je_n(h, \cosh u)$ and $Jo_n(h, \cosh u)$ are even and odd first-kind radial Mathieu functions, $Se_n(h, \cos v)$ and $So_n(h, \cos v)$ are the even and odd first-kind circumferential Mathieu functions, respectively.

The other terms in the above equations are defined as follows:

$$Me_n(h) = \int_0^{2\pi} Se_n^2(h, \cos v)\, dv$$

$$Mo_n(h) = \int_0^{2\pi} So_n^2(h, \cos v)\, dv$$

$$ISe_n(h, v_i, \Delta_i, u_1) = \int_{v_i-\Delta_i}^{v_i+\Delta_i} Se_n(h, \cos v)[\sinh^2 u_1 + \sin^2 v]^{1/2}\, dv$$

$$ISo_n(h, v_i, \Delta_i, u_1) = \int_{v_i-\Delta_i}^{v_i+\Delta_i} So_n(h, \cos v)[\sinh^2 u_1 + \sin^2 v]^{1/2}\, dv$$

$$\Delta_i \cong \tan^{-1}\left[\frac{W_i}{2l\sqrt{\cosh^2 u_1 \cos^2 v_i + \sinh^2 u_1 \sin^2 v_i}}\right]$$

and

$$h = k_c l$$

where W_i is the width of the ith port, k_c is the wavenumber in the substrate l is the semifocal length of the elliptic patch, and u_1 is the corrdinate at the periphery of the path. The Mathieu functions are computed using their infinite series [11]–[14].

The fringing fields are accounted for by the introduction of the effective parameters as follows. The effective relative permittivity [15] of the elliptic patch can be taken as

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left\{1 + \frac{10d}{2b}\right\}^{-1/2}. \quad (3)$$

The effective semiminor axis b_e of the elliptic patch can be taken as [16]

$$b_e = b\left[1 + \frac{2d}{\varepsilon_r \pi b}\left\{\ln\left(\frac{b}{2d}\right) + (1.41\varepsilon_r + 1.77) + \frac{d}{b}(0.268\varepsilon_r + 1.65))\right\}\right]^{1/2} \quad (4)$$

where b is the semiminor axis, given by $b = l \sinh u$ and d is the thickness of the substrate.

Once the Z-matrix of the elliptic disk Z has been determined, S-parameters of the quadrature hybrid are evaluated using the expression:

$$S = (Z - Z_o)(Z + Z_o)^{-1}$$

where Z_o is the characteristic impedance of the transmission lines connected to the device.

For a four-port quadrature hybrid, it is necessary that symmetry is maintained along the x and y axes. This allows us to evaluate the S-parameters using eigenimpedances without involving any matrix inversion. The four eigenvectors are then given in the usual way as [17]

$$\begin{aligned} Z_{e1} &= Z_{11} + Z_{12} + Z_{13} + Z_{14} \\ Z_{e2} &= Z_{11} - Z_{12} + Z_{13} - Z_{14} \\ Z_{e3} &= Z_{11} + Z_{12} - Z_{13} - Z_{14} \\ Z_{e4} &= Z_{11} - Z_{12} - Z_{13} + Z_{14}. \end{aligned} \quad (5)$$

The eigenreflections are then related to the eigenimpedances by

$$S_{ei} = \frac{Z_{ei} - 1}{Z_{ei} + 1}, \qquad i = 1, 2, 3, 4. \quad (6)$$

Finally, the S-parameters are obtained by

$$\begin{aligned} S_{11} &= \tfrac{1}{4}(S_{e1} + S_{e2} + S_{e3} + S_{e4}) \\ S_{12} &= \tfrac{1}{4}(S_{e1} - S_{e2} + S_{e3} - S_{e4}) \\ S_{13} &= \tfrac{1}{4}(S_{e1} + S_{e2} - S_{e3} - S_{e4}) \\ S_{14} &= \tfrac{1}{4}(S_{e1} - S_{e2} - S_{e3} + S_{e4}). \end{aligned} \quad (7)$$

III. Theoretical and Experimental Results

A. Basic Structure

The two-fold symmetry requirement of a quadrature hybrid necessitates that

$$v_2 = -v_1, \quad v_3 = \pi + v_1, \quad \text{and} \quad v_4 = \pi - v_1$$

where v's are the port angles in elliptic coordinates.

Now, for a given microstrip substrate the design variables for the elliptic patch are: eccentricity e, major axis a, and port angle v_1. The basis for determining the parameters are that the waves from the coupled ports be virtually equal over a frequency band. If one achieves this by varying the eccentricity and the port angle, then the isolation and match can be improved by connecting matching networks at the physical ports. The operational center frequency is essentially determined by the major axis of the elliptic patch.

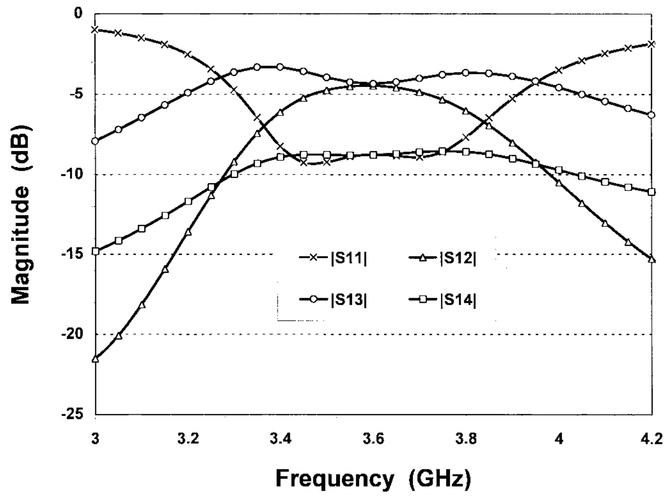

Fig. 2. Theoretical results for the S-parameters of the elliptic-patch quadrature hybrid without matching network.

As the circular patch is really a special case of the elliptic patch (i.e., $e = 0$), the circular patch values [3] can be used as an initial starting point for design values for an elliptic patch with very small eccentricity ($e = 0.05$), then e, a, and v_1 are varied until a satisfactory response of the quadrature hybrid is obtained. This is possible as the analytical expressions for the elliptic patch given in (1) and (2) are computationally fast.

Fig. 2 shows the theoretical results of a quadrature hybrid using an elliptic patch designed on a RT Duroid 5880 substrate which has relative permittivity of 2.20 and thickness of 0.508 mm. The elliptic patch has eccentricity $e = 0.47$, major axis $a = 35.0$ mm, and port angle $v_1 = 51°$. As expected, the isolation of the junction is not good but it exhibits a virtually equal-power split around the center frequency of 3.6 GHz. Having obtained this basic structure, our next step is simply to improve the match by a suitable matching network.

B. Impedance Step-Matching Network

It is well known that for a four-port symmetrical junction it is not necessary to design for both match as well as isolation. It is sufficient to optimize for best match conditions and the isolation then follows suit.

The impedance-matching network used is simply a section of transmission line attached to the ports. The eigenimpedances at the input of the matching network $Z_{ei,0}$, can be obtained by transforming the eigenimpedances at the periphery of the elliptic patch through

$$Z_{ei,0} = \frac{Z_c(Z_{ei} + jZ_c \tan \beta s_0)}{Z_c + jZ_{ei} \tan \beta s_0}, \qquad i = 1, 2, 3, 4 \quad (8)$$

where Z_{ei} is the eigenimpedence at the periphery of the elliptical patch, Z_c is the line characteristic impedance, s_0 is the line length, and β is the wavenumber of the line.

Note that by putting a section of transmission line to the elliptic patch, we introduce two additional variables to the optimization parameter set, i.e., line length and the characteristic impedance of the transmission line. Now there are five variables in an elliptic-patch quadrature hybrid. We can, however, keep the port angle as $51°$ since it gives a good power split between the coupled ports. Also,

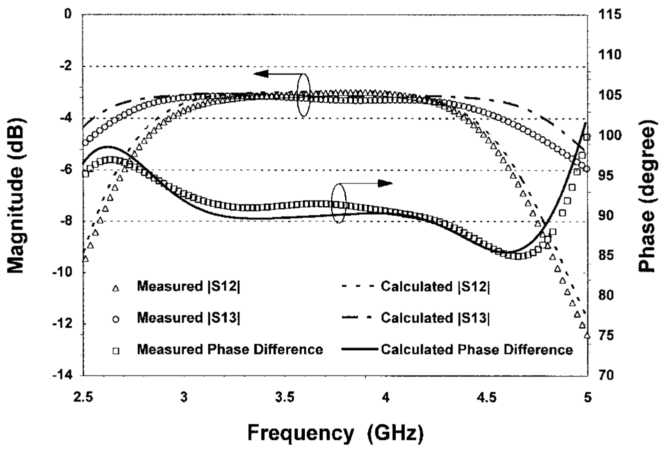

Fig. 3. Theoretical and experimental results at the coupling ports (S_{12} and S_{13}) of the elliptic patch quadrature hybrid with impedance step-matching network. The phase difference is the difference between S_{12} and S_{13}.

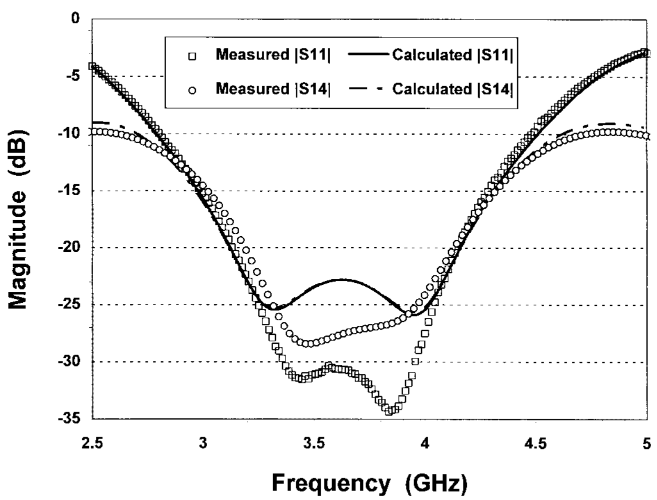

Fig. 4. Theoretical and experimental response of S_{11} and S_{14} of the elliptic-patch quadrature hybrid with impedance step-matching network.

the length of the major axis a is maintained as 35 mm to ensure the device works roughly in the same frequency region. Then the length and the characteristic impedance of transmission lines, together with the eccentricity of the elliptic patch, are varied to improve the match.

It was found fairly rapidly that the best match, and consequently, isolation was obtained when $Z_c = 23.47\ \Omega$, $s_0 = 12.0$ mm, $e = 0.62$, and $a = 39.0$ mm. This device has flat coupling and low VSWR over the frequency range of 3.1–4.1 GHz which gives a bandwidth of about 27%. The bandwidth here is defined as the ratio between the operational center frequency and the operational frequency band with at least -20-dB isolation. It clearly shows significant improvement over similar devices of this kind. The theoretical results are compared with measurements in Figs. 3 and 4. Both sets of results are in overall good agreement; the minor discrepancies between them are probably due to connector effects and tolerances.

Fig. 5 illustrates the variation of worst-case isolation and bandwidth against the change of port width. As can be seen from Fig. 5, the bandwidth and in particular the isolation are rather sensitive to the change of port width. It was found that wider bandwidth can be achieved by increasing the port width at the price of decreasing the isolation. Therefore, a balance between the isolation and bandwidth can be finely adjusted to obtain the optimum response by varying the

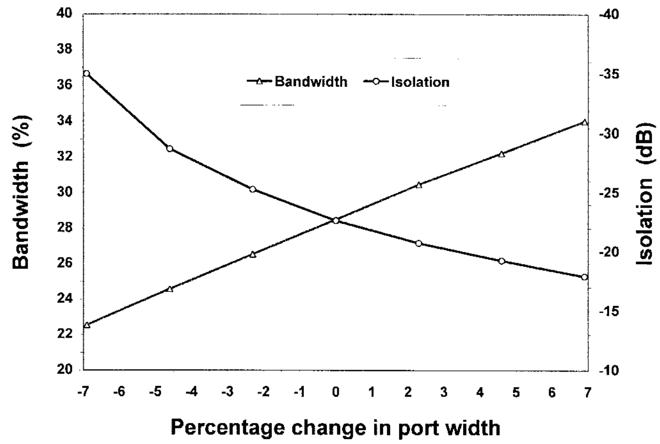

Fig. 5. Variation of bandwidth and isolation against the change of port width. Zero percentage is the optimum width 4.3 mm which corresponds to $Z_c = 23.47\ \Omega$.

width of the matching line.

Other matching configurations such as tapered lines have been investigated and found to be slightly less sensitive to the change of port width, but do not offer a major enhancement to the performance of the device.

IV. Scaling to Higher Frequencies

From (1) and (2), one sees that scaling the physical dimensions of the device for it to operate at a different frequency is not straightforward. In addition, scaling the device also requires the parameters of the matching network to be changed. If, however, we assume that the variation of port angle due to the width of the matching network at the ports Δv is small, then it is possible to derive expressions which give approximate dimensions of the junction to work at other frequencies.

Let the operational center frequency of the quadrature hybrid change from f_o to f_n. The new major-axis length a_n of the elliptic patch and the new line width W_n of the microstrip line at the ports can then be approximated by

$$a_n = \frac{a_o}{x}, \quad W_n = \frac{W_o}{\sqrt{x}} \tag{9}$$

where x is a frequency-scaling factor given by

$$x = \frac{f_n}{f_o}$$

and a_o and w_o are the length of major axis and line width of the device operating at the center frequency f_o.

Equation (9) gives a good initial approximation of the new dimensions required as long as $x < 6$. Some further fine tuning can be applied by changing the eccentricity and port angle in order to obtain optimal response. The power coupling and isolation is then taken care of as outlined previously.

As an example, consider an elliptic-patch quadrature hybrid operating at the center frequency around 14.4 GHz, i.e., $x = 4$ for $f_o = 3.6$ GHz. The initial dimensions given by (9) are $a_n = 9.75$ mm, $W_n = 2.15$ mm. After some adjustments, optimal performance was found when $a = 10.48$ mm, $W = 2.06$ mm, $e = 0.66$ and $s_o = 10.0$ mm. The difference between the dimensions given by (9) and the final dimensions is less than 10% and this suggests that (9) gives a reasonably good approximation. Fig. 6 shows the theoretical frequency response of the device which has an operational bandwidth of 22%.

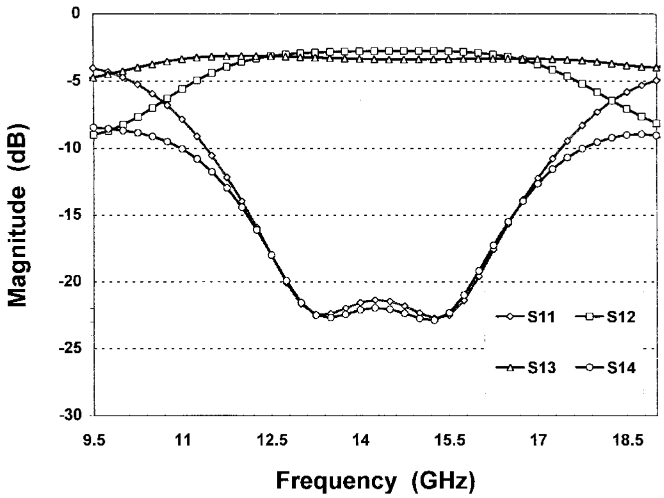

Fig. 6. Theoretical results for the S-parameters of the elliptic-patch quadrature hybrid with impedance step-matching network and operating at the center frequency of 14.4 GHz.

V. Conclusion

A four-port quadrature hybrid with flat-coupling response and operational bandwidth of over 25% has been obtained by using an elliptic patch and a matching network. The use of an elliptic patch as a quadrature-hybrid junction is shown to have superior performance than available designs, especially at higher frequencies where transmission-line equivalents have serious problems. Designing the hybrid for other frequencies can start out in the manner outlined in this paper or can use the scaling expression. In either case, some fine tuning of the parameters when performing the numerical design is required.

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